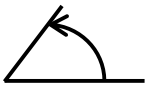
Geometry (Part 1)

Lines and angles

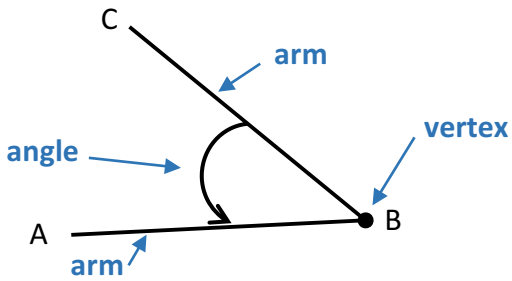
A **line** is an infinite number of points between two end points.

Where two lines meet or cross, they form an **angle**.

An **angle** is an amount of rotation. It is measured in **degrees**.

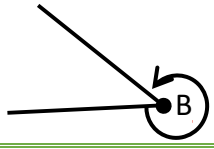
Types of angles		
Name of angle	**Example**	**Size of angle**
Acute angle		Between $0°$ and $90°$
Right angle		Equal to $90°$
Obtuse angle		Between $90°$ and $180°$
Straight line		Equal to $180°$
Reflex angle		Between $180°$ and $360°$
Revolution/angles around a point		Equal to $360°$

Angle language:

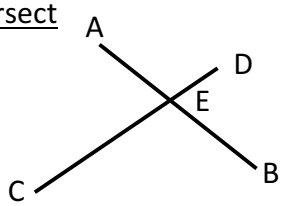
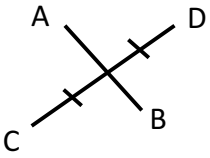
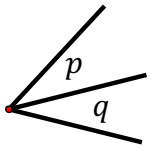
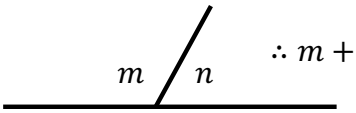
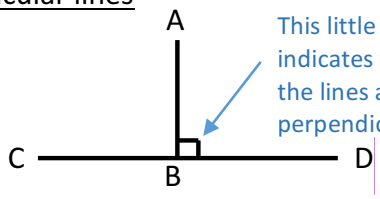


Labelling angles: $\hat{B}$ or $A\hat{B}C$

Also:



We refer to the reflex angle as **'reflex $\hat{B}$'**

Terminology	
Intersect 	AB and CD intersect (cross or cut) at E
Bisect 	AB bisect (cuts in half) CD
Complementary angles Angles that add up to $90°$ Supplementary angles Angles that add up to $180°$	E.g. the complement of $48°$ is $42°$ E.g. the supplement of $130°$ is $50°$
Adjacent angles  Adjacent angels on a straight line adds up to $180°$  $\therefore m + n = 180°$	Angles that have a common vertex and a common arm → p and q are adjacent angles.
Perpendicular lines  This little block indicates to us that the lines are perpendicular.	Lines that meet or cross at $90°$. $AB \perp CD$ Symbol for 'perpendicular'

Exercise 1:

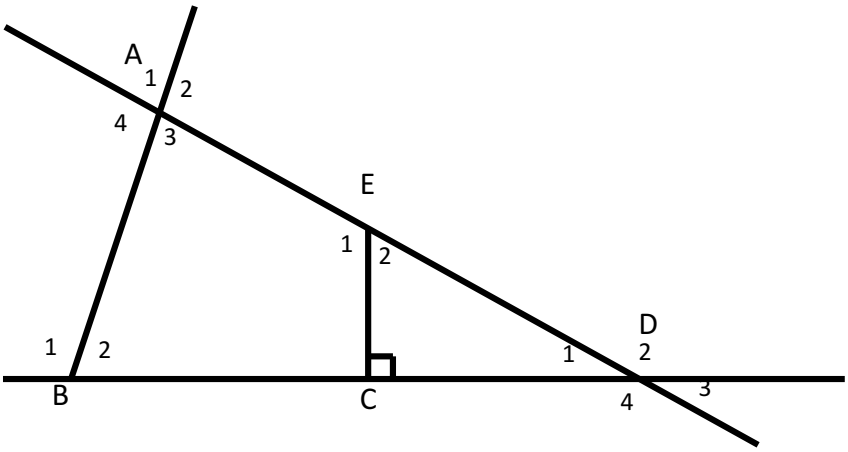
(a) In the diagram below name:

(1) 5 acute angles

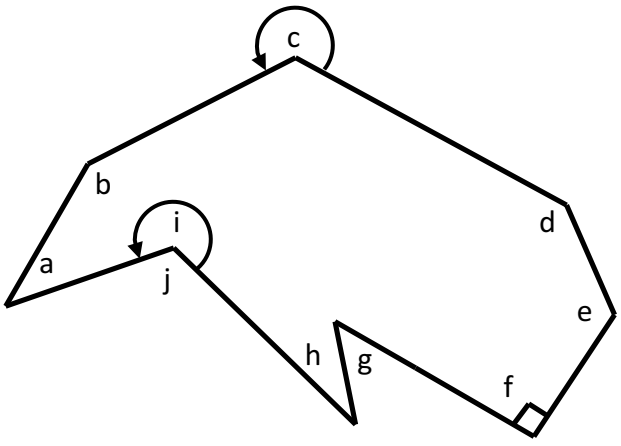
(2) 2 right angles

(3) 10 pairs of adjacent angles

(4) 3 obtuse angles



(b) In the diagram below, classify the angles labelled a – j. The first one is done for you as an example:



a: Acute b: _____

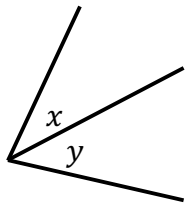
c: _____ d: _____

e: _____ f: _____

g: _____ h: _____

i: _____ j: _____

(c) Consider the angles marked x and y. State whether they are adjacent or not:



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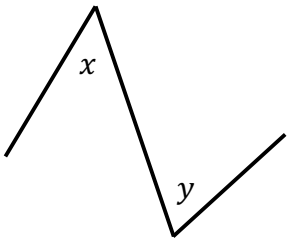

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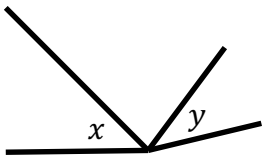

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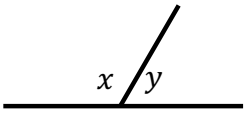

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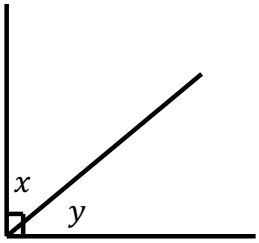


_____

_____

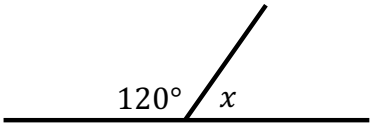
(d) Complete the table by filling in the missing information:

Measure of angle	Complement	Supplement
$37°$	$90° - 37° = 59°$	$180° - 37° = 143°$
$20°$		
$77°$		
$101°$		
$90°$		
$96°$		
x		
y		

REMEMBER: Adjacent angles on a straight line are supplementary.

If they are adjacent angles on a straight line, then they add up to $180°$.

Example:
Determine, with reason, the value of x:



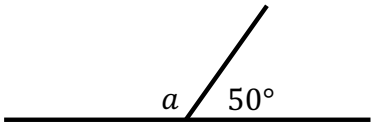
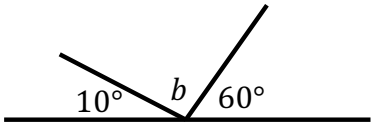
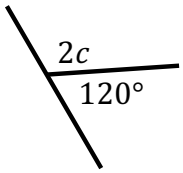
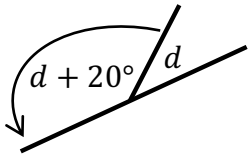
Statement	Reason
$x = 180° - 120°$	Adj ∠'s on a str line

We use these abbreviations to make our lives a little bit easier! *there is a complete summary on page

In geometry we always need to provide **reasons** for 'why' we state something.

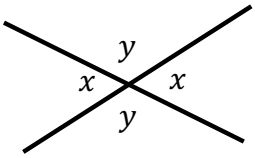
Exercise 2:

Calculate the size of the variables (a, b, c and d). Give a reason for your answer.

		Statement	Reason
(a)			
(b)			
(c)			
(d)			

Vertically opposite angles:

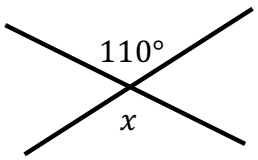
When two straight lines intersect the angles opposite each other are called **vertically opposite angles.**



Vertically opposite angles are equal to each other.

Example:

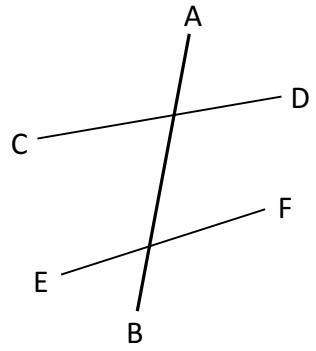
Determine, with reason, the value of x:



Statement	Reason
$x = 110°$	Vert opp ∠'s

Transversals

If a line **cuts or touches** another line, it is called a **transversal**.

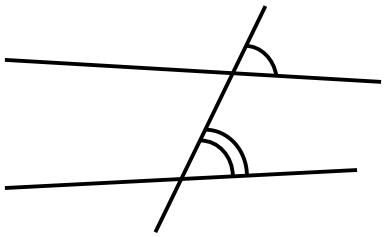


e.g. AB is a transversal because it cuts CD and EF, CD and EF are also transversals of AB.

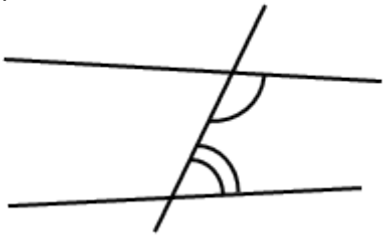
Transversals creates three important types of angles, namely:

1. Corresponding angles
2. Co-interior angles
3. Alternating angles

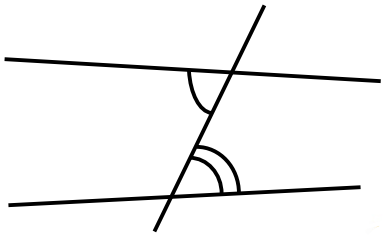
1. Corresponding angles are in the *same position* as each other. They make a F shape:



2. Co-interior angles are *between the lines* and *on the same side* of the transversal. They are "*inside together*". They make a C or U shape.



3. Alternate angles are *between the lines* and on *alternate (opposite) sides* of the transversal. They make a Z or N shape.



Remember the word FUN whenever you see a transversal!

Exercise 3:

Use the diagram below to find:

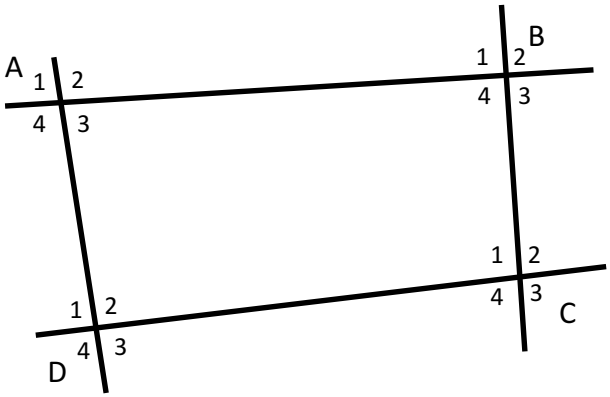
(a) 10 pairs of corresponding angles

(b) 8 pairs of vertically opposite angles

(c) 4 pairs of co-interior angles

(d) 8 pairs of alternate angles

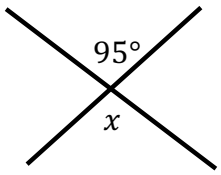
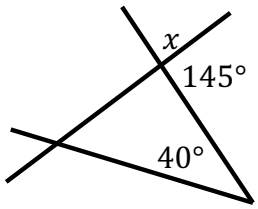
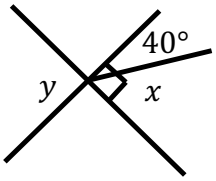
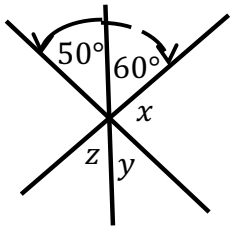
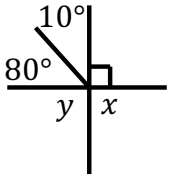
(e) 6 pairs of adjacent supplementary angles



Use the following reasons to help you complete Ex 4 and 5
- Adj ∠'s on a str Line
- Adj comp ∠'s
- Vert opp ∠'s
- ∠'s at a pt

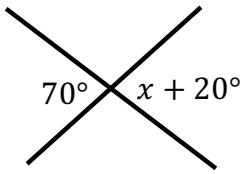
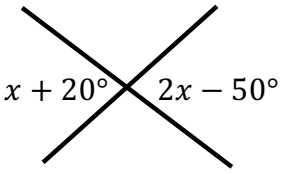
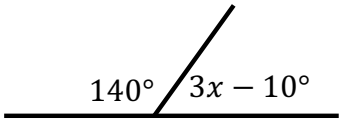
Exercise 4:

Find the value of each variable, in alphabetical order (where there is more than one variable), providing reasons for your statements:

		Statement	Reason
(a)			
(b)			
(c)			
(d)			
(e)			

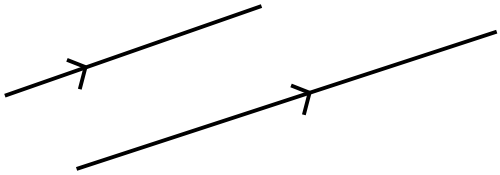
Exercise 5:

Use the diagram to write down an equation, with a reason, in order to calculate the value of x:

		Statement	Reason
(a)			
(b)			
(c)			

Parallel lines

Parallel lines are lines that stay the **same distance apart**, no matter how long the lines are (they are lines that never meet).



Arrows are used to indicate that lines are

If lines are **parallel** then:

- The **corresponding angles are equal**
- The **alternate angles are equal**
- The **co-interior angles are supplementary**

Reasons:
corr ∠'s ; …//…
alt ∠'s ; …//…
co-int ∠'s ; …//…

NB: You have to mention the parallel lines

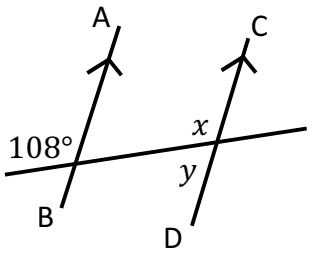
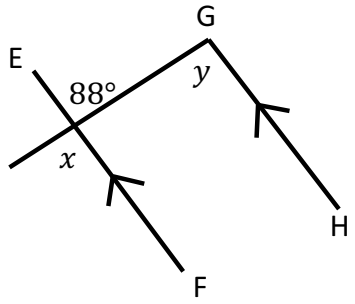
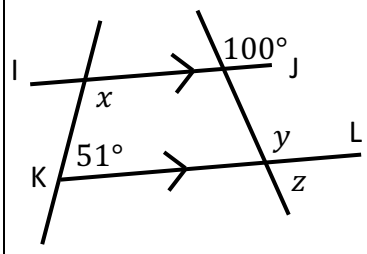
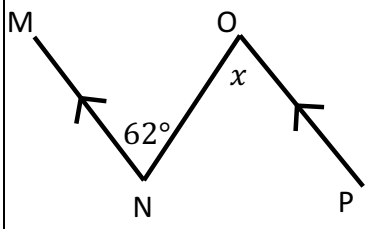
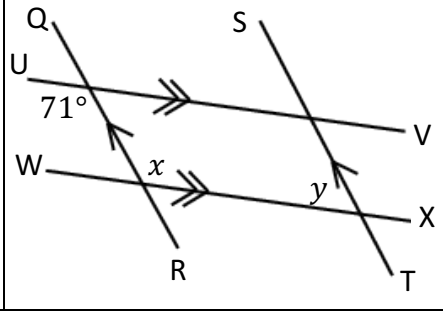
To prove lines are parallel:

Prove the corresponding angles are equal	corr ∠'s =
Prove the alternate angles are equal	alt ∠'s =
Prove the co-interior angles are supplementary	co-int ∠'s =

Let's see in Exercise 6 how these parallel lines can help us determine the value of unknown angles…

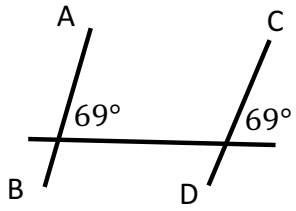
Exercise 6:

(a) Determine the sizes of the angles marked with variables, in alphabetical order. Give reasons for your answers. (The first one is done for you as an example)

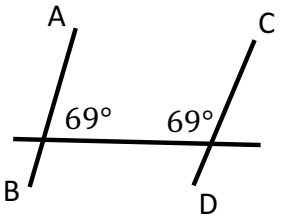
		Statement	Reason
(1)		$x = 108°$ $y = 180 - 108°$ $y = 72°$	Corr ∠'s ; AB//CD Adj ∠'s on a str line
(2)			
(3)			
(4)			
(5)			

(b) In each case, state whether AB is parallel to CD. Provide reasons for your statements.

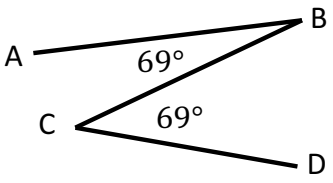
(1)



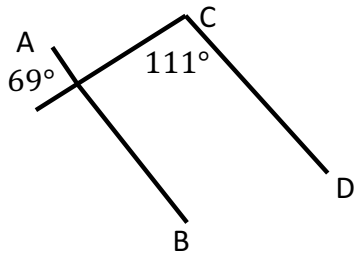
(2)



(3)



(4)



Summary of statements and reasons	
Statement	**Reason**
Angles on a straight line adds up to $180°$	Adj $\angle$'s on a str line
Complementary angles adds up to $90°$	Adj comp $\angle$'s
Vertically opposite angles are equal	Vert opp $\angle$'s
Angles around a point adds up to $360°$	$\angle$'s at a pt
Corresponding angles of parallel lines are equal	Corr $\angle$'s ; …//…
Co-interior angles between parallel lines add up to $180°$	Co-int $\angle$' ; …//…
Alternating angles of parallel lines are equal	Alt $\angle$'s ; …//…

**Please note that none of the diagrams in this workbook are drawn according to scale.*

MEMO

Exercise 1:

(a.1) $\hat{A}_1$; $\hat{A}_3$; $\hat{E}_2$; $\hat{D}_1$; $\hat{D}_3$; $\hat{B}_2$ (any five)

(a.2) $E\hat{C}B$ and $E\hat{C}D$

(a.3) $\hat{A}_1$ and $\hat{A}_2$; $\hat{A}_2$ and $\hat{A}_3$; $\hat{A}_3$ and $\hat{A}_4$; $\hat{B}_1$ and $\hat{B}_2$; $E\hat{C}B$ and $E\hat{C}D$; $\hat{D}_1$ and $\hat{D}_2$; $\hat{D}_2$ and $\hat{D}_3$; $\hat{D}_3$ and $\hat{D}_4$ $\hat{A}_1$ and $\hat{A}_4$; $\hat{D}_1$ and $\hat{D}_4$; $\hat{E}_1$ and $\hat{E}_2$

(a.4) $\hat{A}_2$; $\hat{A}_4$; $\hat{E}_1$; $\hat{D}_2$; $\hat{D}_4$; $\hat{B}_1$ (any three)

(b) b: Obtuse
c: Reflex
d: Obtuse
e: Obtuse
f: Right
g: Acute
h: Acute
i: Reflex
j: Obtuse

(c.1) Adjacent

(c.2) Not adjacent (does not share a common point)

(c.3) Not adjacent (does not share a common arm)

(c.4) Adjacent

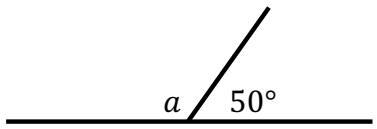
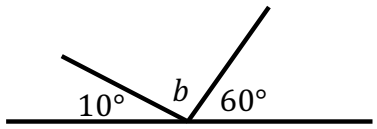
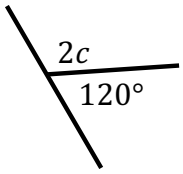
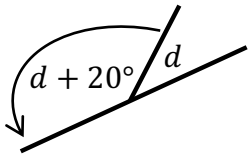
(c.5) Adjacent

(c.6) Not adjacent (does not share a common point)

(d)

Measure of angle	Complement	Supplement
$20°$	$70°$	$160°$
$77°$	$13°$	$103°$
$101°$	No complement	$79°$
$90°$	$0°$	$90°$
$96°$	No complement	$84°$
x	$90° - x$	$180° - x$
y	$90° - y$	$180° - y$

Exercise 2:

		Statement	Reason
(a)		$a = 180° - 150°$ $\therefore a = 130°$	Adj ∠'s on a str line
(b)		$b = 180° - 10° - 60°$ $\therefore b = 110°$	Adj ∠'s on a str line
(c)		$2c = 180° - 120°$ $2c = 60°$ $c = \frac{60°}{2}$ $\therefore c = 30°$	Adj ∠'s on a str line
(d)		$d + 20° + d = 180°$ $2d = 180° - 20°$ $2d = 160°$ $d = \frac{160°}{2}$ $\therefore d = 80°$	Adj ∠'s on a str line

Exercise 3:

(a) $\hat{A}_1$ and $\hat{B}_1$; $\hat{A}_2$ and $\hat{B}_2$; $\hat{A}_3$ and $\hat{B}_3$; $\hat{A}_4$ and $\hat{B}_4$; $\hat{A}_1$ and $\hat{D}_1$; $\hat{A}_2$ and $\hat{D}_2$; $\hat{A}_3$ and $\hat{D}_3$; $\hat{A}_4$ and $\hat{D}_4$
$\hat{B}_1$ and $\hat{C}_1$; $\hat{B}_2$ and $\hat{C}_2$; $\hat{B}_3$ and $\hat{C}_3$; $\hat{B}_4$ and $\hat{C}_4$; $\hat{C}_1$ and $\hat{D}_1$; $\hat{C}_2$ and $\hat{D}_2$; $\hat{C}_3$ and $\hat{D}_3$; $\hat{C}_4$ and $\hat{D}_4$
(any ten pairs)

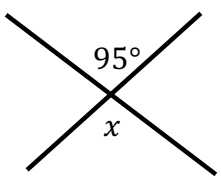
(b) $\hat{A}_1$ and $\hat{A}_3$; $\hat{A}_2$ and $\hat{A}_4$; $\hat{B}_1$ and $\hat{B}_3$; $\hat{B}_2$ and $\hat{B}_4$; $\hat{C}_1$ and $\hat{C}_3$; $\hat{C}_2$ and $\hat{C}_4$; $\hat{D}_1$ and $\hat{D}_3$; $\hat{D}_2$ and $\hat{D}_4$

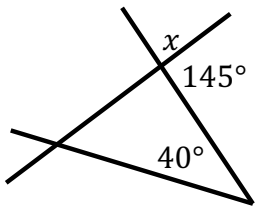
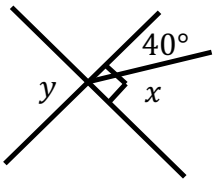
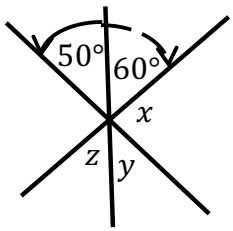
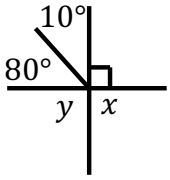
(c) $\hat{A}_3$ and $\hat{D}_2$; $\hat{A}_4$ and $\hat{D}_1$; $\hat{A}_2$ and $\hat{B}_1$; $\hat{B}_4$ and $\hat{C}_1$; $\hat{B}_3$ and $\hat{C}_2$; $\hat{C}_1$ and $\hat{D}_2$; $\hat{C}_4$ and $\hat{D}_3$
(any four)

(d) $\hat{A}_2$ and $\hat{B}_4$; $\hat{A}_4$ and $\hat{D}_2$; $\hat{A}_3$ and $\hat{D}_1$; $\hat{B}_1$ and $\hat{A}_3$; $\hat{B}_4$ and $\hat{C}_2$; $\hat{B}_3$ and $\hat{C}_1$; $\hat{C}_1$ and $\hat{D}_3$; $\hat{C}_4$ and $\hat{D}_2$

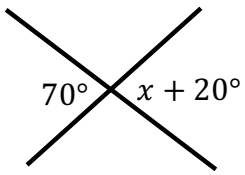
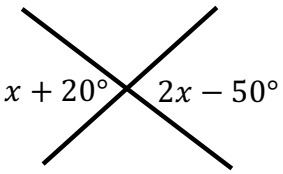
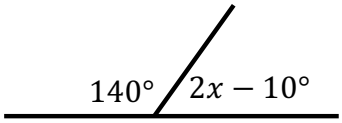
(e) Any two angles that are on a straight line and share the same point.

Exercise 4:

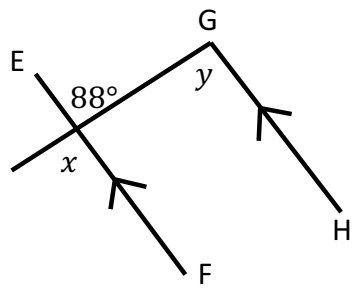
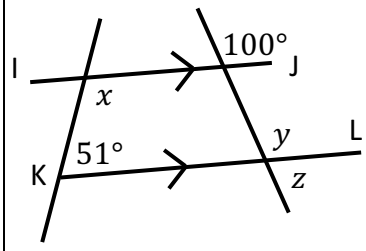
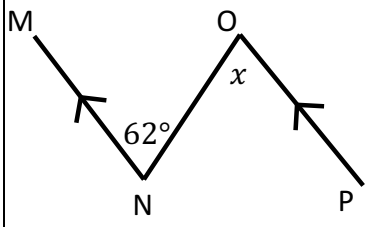
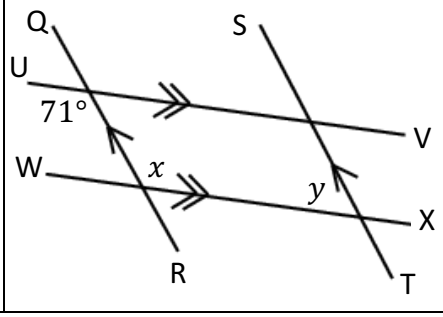
		Statement	Reason
(a)		$x = 95°$	Vert opp ∠'s

(b)		$x = 180° - 145°$ $\therefore x = 35°$	Adj $\angle$'s on a str line
(c)		$x = 90° - 40°$ $\therefore x = 50°$ $y = 90°$	Adj comp $\angle$'s
(d)		$x + 50° + 60° = 180°$ $x = 180° - 50° - 60°$ $\therefore x = 70°$ $y = 50°$ $z = 60°$	Adj $\angle$'s on a str line Vert opp $\angle$'s Vert opp $\angle$'s
(e)		$x = 90°$ $y = 90°$	Adj $\angle$'s on a str line Vert opp $\angle$'s

Exercise 5:

		Statement	Reason
(a)		$70° = x + 20°$ $\therefore x = 50°$	Vert opp $\angle$'s
(b)		$x + 20° = 2x - 50°$ $20° + 50° = x$ $70° = x$ $\therefore x = 70°$	Vert opp $\angle$'s
(c)		$2x - 10° + 140° = 180°$ $2x + 130° = 180°$ $2x = 50°$ $x = 25°$	Adj $\angle$'s on a str line

Exercise 6:

		Statement	Reason
(2)		$x = 88°$ $y = 88°$	Vert opp ∠'s Corr ∠'s ; EF // GH
(3)		$x + 51° = 180°$ $\therefore x = 129°$ $y = 100°$ $z = 180° - 100°$ $\therefore z = 80°$	Co-int ∠'s ; IJ // KL Corr ∠'s ; IJ // KL Adj ∠'s on a str line
(4)		$x = 62°$	Alt ∠'s ; MN // OP
(5)		$x = 71°$ $y + 71° = 180°$ $\therefore y = 109°$	Alt ∠'s ; UV // WX Co-int ∠'s ; QR // ST

(b.1) AB // DC because corresponding angles are equal.

(b.2) AB will not be parallel to DC because the co-interior angles are not supplementary.

(b.3) AB // DC because the alternating angles are equal.

(b.4) AB // DC because the co-interior angles will be supplementary.